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Rover Operations and Dynamics of Dust – An Analytical Approach using Lambert *W*-Function

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Abstract

A proposition wherein the dynamics of dust during mini-rover operations on lunar surface taking into account of probable resistance offered by an equilibrium local concentration of gases of near-surface lunar atmosphere can be best described by considering a weak linear drag force in the equations of motion is presented. The equations of motion vis-à-vis the one considering a vacuum near-surface lunar atmosphere are transcendental in nature whose exact closed-form solutions can be obtained using Lambert W-function only. An analysis is made giving specific consideration of Chandrayaan-2 Rover and results are presented.

Keywords : *Lambert W-function; Chandrayaan-2 Lunar Rover; Lunar dust.*

1. INTRODUCTION

The lunar surface is known to be covered by a thick blanket of fine dust. The lunar "regolith" is a blanket of soil and rocks 3 to 20 m thick that covers the lunar surface. The size of lunar soil particles varies from 50 μm to 1 mm in diameter[1]. Through various lunar soil mechanics experiments, it is reported that lunar soil exhibits strong cohesive forces. The predominant forces between lunar particles are van der Waals and electrostatic interactions. The lunar dust consists of extremely fine and electrostatically charged particles. Lunar particle cohesion has been estimated to be roughly 0.035 to 0.05 N/cm^2 from lunar surface experiments[2]. Lunar dust clings tenaciously to surfaces and during Apollo missions, adhesive forces, were observed to vary roughly from 0.01 to 0.1 N/cm^2 and deposition of such dust on sensitive surfaces like solar panel, optical lenses etc. causes serious problems [3]. In this regard, an analysis is carried out to understand the dynamics of dust under the equilibrium near-surface lunar atmosphere during mini rover operations giving specific consideration of Chandrayaan-2 rover and results are reported.

2. ROVER OPERATIONS AND ROVER WHEEL MECHANICS

During the rover operations on the surface of Moon, dust will get collected on the rover wheel and subsequently, it will either remain stuck to the wheel or be kicked off following the trajectory given by the dynamics of rover wheel depending on the mode of rover operation. The modelled analysis of the dynamics of dust can be studied under the following cases:

i. Normal rover operation (i.e. rolling wheel); &
ii. Contingency rover operation (i.e. slipping wheel).

2.1 NORMAL ROVER OPERATION

During the normal mode of rover operation, rover wheels roll on the surface. Motion of a rolling wheel can be considered as the combination of pure rotational motion of the wheel and pure translation of the wheel about the centre of the rover wheel. Let us consider a rover wheel of radius "R" rotating with an angular velocity of "ω" as shown in Fig. 1. Let the velocity of the centre of the wheel be "V_{CM}". Then, the condition of pure rolling gives

$$V_{CM} = \omega R \qquad (1)$$

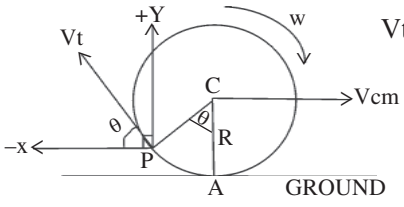

Fig. 1

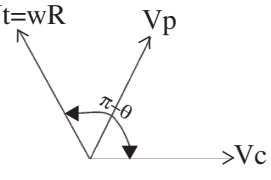

Fig. 2

The velocity of any point "P" on the circumference of the wheel is different from V_{CM} and can be calculated as follows as shown in fig. 2. A rolling point "P" is acted by two component velocities; one component velocity V_t due to rotation and other component velocity V_c due to translation. Thus, the velocity of the point "P" can be written as $\vec{V}_P = \vec{V}_C + \vec{V}_t$. Pure rolling condition yields: $\left|\vec{V}_t\right| = \omega R = \left|\vec{V}_{CM}\right| = \left|\vec{V}_C\right|$

$$\Rightarrow V_P = \left|\vec{V}_P\right| = 2V_{CM}\sin\left(\frac{\theta}{2}\right) \quad (2)$$

Thus, a dust particle collected at "P" attains the velocity given by (2) and will tend to fly tangentially along a trajectory which will be subsequently shown to follow the equation (10). To study the trajectory of the dust, the resultant velocity given by equation (2) can be resolved into two orthogonal axes defined as shown in fig. 1. Thus, it can be written as

$\vec{V}_P = \vec{V}_{PX} + \vec{V}_{PY}$; where

$$\vec{V}_{PX} = \vec{u}_x = V_{CM}(\hat{x}) + V_{CM}\cos\theta(-\hat{x}) = V_{CM}(1-\cos\theta)\hat{x}$$

$$\vec{V}_{PY} = \vec{u}_y = V_{CM}\sin\theta(\hat{y}) \quad (3)$$

Now, to find the direction of the resultant velocity at "P", let "á" be the angle made by the resultant velocity "V_P" with the horizontal direction. Then,

$$\tan\alpha = \frac{\left|\vec{u}_y\right|}{\left|\vec{u}_x\right|} = \cot\left(\frac{\theta}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) \Rightarrow \alpha = \frac{1}{2}(\pi - \theta)$$

i.e. resultant velocity at "P" bisects always the angle $(\pi - \theta)$.

2.2 CONTINGENCY ROVER OPERATION

During this rover operation mode, wheel(s) of rover is (are) assumed to get stuck on (dusty) ground so that rover wheel is in slip operation mode. In other words, rover wheel is experiencing only 100% rotation and 0% translation. Since there is no translational motion of the wheel, the velocity of a point "P" is given by

$$\Rightarrow V_P = V_t = \omega R \quad (4a)$$

Thus, unlike a rolling wheel, all the circumferential points on a slipping wheel will have the same velocity whose magnitude is given by equation (4a). Thus, a dust particle adhered at P will tend to fly tangentially with initial velocity given by (4a) along a trajectory as derived in equation (10).

The components of the resultant velocity obtained in equation (4a), resolving along the two orthogonal axes as defined in fig. 1, are given by

$$\vec{V}_{PY} = \vec{u}_y = \omega R\sin\theta(+\hat{y}) \text{ and } \vec{V}_{PX} = \vec{u}_x = \omega R\cos\theta(-\hat{x}) \quad (4b)$$

3. DUST DYNAMICS

It is a well known fact that the acceleration due to gravity on the surface of the Moon is about one-sixth of that on Earth: roughly 1.63 m/s^2. This is too weak to retain an atmosphere like that of Earth. However, it is capable of retaining a thin atmosphere which consists of traces of Ne, He, H_2, and Ar, and even smaller amounts of CO_2 and H_2O. Some of these gases are trapped in the soil during the cold lunar night and escape as the surface heats up from the sun. Gas abundances of all the constituents of lunar atmosphere and some scale heights in the lunar atmosphere as well as the variations in abundances of the four principal lunar atmospheric gases (Ne, He, H_2, and Ar) over day/night cycles at different latitudes and longitudes are available[4]. The daytime undisturbed gas concentrations (molecules/cm^3) of the four major constituents of lunar atmosphere viz. ^{20}Ne, He, H_2, and ^{40}Ar are respectively ~4x10^3 to 10^4 ; ~8x10^2 to 4.7x10^3 ; ~2.5-9.9x10^3; and ~2x10^3 [4]. The respective numbers during night time are 10^5; 4 to 7x10^4; 10^4 to 1.5x10^5; and <10^2 [4]. Also, density of gases decreases with increase in altitude. Thus, these local concentrations of trace gases may be assumed to offer resistance to the motion of minute lunar dust particles during mini-rover operations like Chandrayaan-2 rover of height less than a meter. Thus, the dynamics of dust during rover operations on lunar surface taking into account of probable resistance offered by an equilibrium local concentration of gases of near-surface lunar atmosphere might be best described by considering a weak drag force in the equations of motion., The weak drag force can be assumed as a drag force which is linear speed and its general form can be written as $\vec{f}_d = -b\vec{v}$, where "b" is a constant whose value depends on the properties of the medium and on the shape & dimensions of the object. For a spherical object, the constant "b" takes Stoke's constant $6\pi\eta_d r$, where r is the radius of the sphere and η_d is the dynamic viscosity of the fluid (medium).

To determine whether linear or quadratic drag applies to a system, one must calculate the Reynold's number, $\Re_e$. Such a linear (in speed) drag force actually occurs for low Reynold's number, $\Re_e < 1$ and represents a streamline flow. For $1 << \Re_e << 10^5$, the motion is turbulent and quadratic drag applies. Quadratic drag is suitable for motion of dust particles in Earth's atmosphere with thick atmosphere and also in Mars atmosphere wherein the dynamics of dust is very much affected by the wind velocity which varies from 2 m/s to 10 m/s near the surface[5]. The motion of lunar dust, on the other hand, is mainly gravity driven. Though lunar dust are available in different textures, lunar dust particles are assumed to be spherical for simplicity and average lunar particle density is taken to be 3.01 gm/cm^3[3]. Then, for a spherical dust of radius r moving with speed v in the local cloud of gases (fluid) of density ρ_d and dynamic viscosity η_d (or kinematic viscosity η_k), the Reynold's number and drag force are given by

$$\Re_e = \frac{2rv}{\eta_k} = \frac{2r\rho_f v}{\eta_d} \tag{5a}$$

$$f_d = 6\pi\eta_d rv \quad for \quad \Re_e < 1 \tag{5b}$$

The terminal speed v_t is then given by

$$6\pi\eta_d r v_t = mg_m = \frac{4}{3}\pi r^3 \rho_s g_m \tag{5c}$$

where ρ_s is the density of a spherical dust and g_m is the lunar gravity. In order to apply equation (5a) to the whole of the trajectory, we require $\Re_e \le 1$ for speeds up to the terminal speed. For solving equation (5c) for speed up to terminal speed i.e. $v \to v_t$, the radius of the spherical lunar dust is found to be

$$r = \left[\frac{9\Re_e \eta_d^2}{4\rho_f \rho_s g_M}\right]^{1/3} \tag{5d}$$

Assuming motion of a lunar dust in a local concentration of gases of each of the major constituents of near-surface lunar atmosphere, for $\Re_e \le 1$, one finds that equation (5d) applies when

Radius (μm)	Ne	He	Ar
r	≤ 8.18	≤ 101.8	≤ 51.6

Spheres this small is of our interest as scale of lunar dust size is of the order of these spheres. Thus, linearity of drag force may be considered in the equations of motion.

3.1 VECTOR EQUATIONS OF MOTION

Defining the +Y-axis along the upward vertical direction, and +X-axis along the horizontal right direction as shown in fig.1, the equations of motion of a dust particle of mass "m" at point "P" can be written as follows:

$$m\frac{dv_x}{dt} = -bv_x \text{ and } m\frac{dv_y}{dt} = -mg - bv_y \tag{6}$$

The first order (in velocity) uncoupled linear differential equations (6) can be effectively combined in a single vector equation (7a) as follows:

$$m\frac{d\vec{v}(t)}{dt} = \vec{F} + \vec{f}_d \tag{7a}$$

Where the gravity force F and drag force f_d are given by

$$\vec{F} = -mg\hat{y} \text{ and } \vec{f}_d = -b\vec{v} \tag{7b}$$

$$\Rightarrow m\frac{d\vec{v}(t)}{dt} = \vec{F} - b\vec{v}(t) \tag{7c}$$

This non-homogeneous first order differential equation is the desired equation whose solution is sought. And it can be solved as follows:

$$m\frac{d\vec{v}(t)}{dt} = -b\left[\vec{v} - \frac{\vec{F}}{b}\right] \tag{7d}$$

Change of variable: $\vec{u} = \vec{v} - \frac{\vec{F}}{b}$ reduces equation (7d) to:

$$m\frac{d\vec{u}}{dt} = -b\vec{u} \tag{7e}$$

The solution of equation (7e) is given by $\vec{u} = \vec{A}e^{-\frac{b}{m}t}$

$$\Rightarrow \vec{v} = \frac{\vec{F}}{b} + \vec{A}e^{-\frac{b}{m}t} \tag{7f}$$

Evaluation $\vec{A}$ using the initial condition i.e. t = 0; v = u, equation (7f) becomes

$$\Rightarrow \vec{v} = \frac{d\vec{r}(t)}{dt} = \frac{\vec{F}}{b} + \left(\vec{u} - \frac{\vec{F}}{b}\right)e^{-\frac{b}{m}t} \tag{8}$$

$$\Rightarrow \vec{r}(t) = \frac{\vec{F}}{b}t - \frac{m}{b}\left(\vec{u} - \frac{\vec{F}}{b}\right)e^{-\frac{b}{m}t} + \vec{B} \tag{9a}$$

Evaluating again $\vec{B}$ using initial condition i.e. t = 0; r = 0, the vector solution of equation (7d) in $\vec{r}$ is obtained

$$\Rightarrow \vec{r}(t) = \frac{\vec{F}}{b}t + \frac{m}{b}\left(\vec{u} - \frac{\vec{F}}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right) \tag{10a}$$

Since $\vec{F} = -mg\hat{y}$; scalar equations of equation (10a) are given by

$$\Rightarrow x(t) = \frac{m}{b}u_x\left(1 - e^{-\frac{b}{m}t}\right) \tag{10b}$$

$$\Rightarrow y(t) = -\frac{mg}{b}t + \frac{m}{b}\left(u_y + \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t}\right) \tag{10c}$$

If "u" is the initial velocity with which a dust particle leaves the rover wheel making an angle "θ" with the horizon, then the dust particle will traverse a trajectory, whose parametric equations are given by equations (10b) and (10c). From equation (10b), the time "t" is obtained as

$$t = -\frac{m}{b}\ln\left(1 - \frac{bx}{mu_x}\right) \tag{11a}$$

Substituting equation (11a) in equation (10c), the equation of complete trajectory of dust particles is given by

$$y(x) = \frac{1}{u_x}\left(u_y + \frac{mg}{b}\right)x + \frac{m^2 g}{b^2}\ln\left(1 - \frac{bx}{mu_x}\right) \tag{11b}$$

Since we get now the parametric equations (10b) and (10c), the dynamics of dust particles can be studied in terms of its time of ascend, maximum height, time of flight, range etc.

3.2 TIME OF ASCEND

Mathematically, time of ascend is defined as the time for which $\dot{y}(t) = 0$. From equation (10c), we get

$$\dot{y}(t) = -\frac{mg}{b} + \left(u_y + \frac{mg}{b}\right)e^{-\frac{b}{m}t}$$

The condition $\dot{y}(t) = 0$ yields the time of ascend t_1 as follows:

$$\Rightarrow \dot{y}(t) = -\frac{mg}{b} + \left(u_y + \frac{mg}{b}\right)e^{-\frac{b}{m}t_1} = 0$$

$$t_1 = \frac{m}{b}\ln\left(1 + \frac{b}{mg}u_y\right) \tag{12}$$

3.3 MAXIMUM HEIGHT

Mathematically, maximum height H is obtained from the condition: $H = y(t_1)$. Thus, from equation (10c), we get

$$\Rightarrow H = -\frac{mg}{b}t_1 + \frac{m}{b}\left(u_y + \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t_1}\right) \tag{13a}$$

Substituting the value of t_1 from equation (12) in equation (13a), we obtain the maximum height H attained by dust particles as follows:

$$H = -g\left(\frac{m}{b}\right)^2 \ln\left(1 + \frac{b}{mg}u_y\right)$$

$$+ \frac{m}{b}\left(u_y + \frac{mg}{b}\right)\left[1 - \exp\left\{-\ln\left(1 + \frac{b}{mg}u_y\right)\right\}\right] \tag{13b}$$

3.4 TIME OF FLIGHT

Mathematically, time of flight is defined as the time for which $y(t) = 0$. Thus, from equation (10c), we get

$$-\frac{mg}{b}t_2 + \frac{m}{b}\left(u_y + \frac{mg}{b}\right)\left(1 - e^{-\frac{b}{m}t_2}\right) = 0$$

$$\Rightarrow e^{-\frac{b}{m}t_2} = -\frac{g}{\left(u_y + \frac{mg}{b}\right)}t_2 + 1 \tag{14}$$

Equation (14) is an implicit equation whose solution cannot be expressed in terms of elementary functions. One has to solve this transcendental equation using numerical techniques or approximations. However, our interest is to solve equation (14) analytically. This is possible only with Lambert *W*-function. This paper presents an approach to use Lambert *W*-function for rover applications and it describes finding the exact closed-form explicit solutions of probable transcendental equations which govern the dynamics of dust in the near-surface trace gases lunar atmosphere during lunar mini-rover operations, using *W*-function.

3.5 LAMBERT *W*-FUNCTION AND ITS OVERVIEW

Lambert *W*-function commonly known as "*W*-function" originated from the work of J. H. Lambert [6] on trinomial equation that he has published in 1758 and was discussed by Euler [7] in 1779. Although rarely used, its properties and algorithms are well documented [8-12]. Nowadays, it is included as an in-built library function in many computing systems such as LambertW[x] in Maple® and ProductLog [x] in Mathematica®. Lambert *W*-function, represented by *W*(z), is defined as the inverse of the function

$$f(z) = ze^z \text{ satisfying } W(z)e^{W(z)} = z \tag{15a}$$

That is: $Y = Xe^X \Leftrightarrow X = W(Y)$ (15b)

The Lambert *W*-function as defined by equation (15) is generally complex-valued. In general, for a given complex number z, equation (15) will have infinite number of solutions and is therefore multivalued function. Thus, in general solutions of equation (15) are written as $W_k(z)$ where the integers k = 0, ±1, ±2, … denote different branches of the Lambert *W*-function.

For our purpose, we are interested in the real-valued W(x). Thus, when z is a real number x, equation (15) can have either two real solutions $W_0(x)$ and $W_{-1}(x)$ for $-1/e \le x < 0$, one real solution $W_0(x)$ for $x \ge 0$, or no real solutions for x<-1/e. By convention, the branch satisfying $W(x) \ge -1$ is denoted by $W_0(x)$ and is defined as the principal branch while the secondary real branch satisfying $W(x) \le -1$ is denoted by $W_{-1}(x)$. The two real branches are shown in fig. 3.

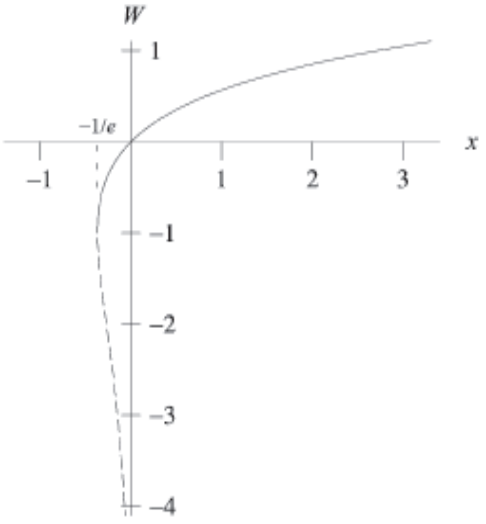

Fig. 3: The two real branches of $W_0(x)$. The solid line shows $W_0(x)$ while the dashed line shows $W_{-1}(x)$. [11]

Some of the values of W-function are given in the following table 1.

Table 1. Some of the values for the Lambert *W*-function. Of the infinite number of branches, only two branches are tabulated.

x	$W_0(x)$	$W_{-1}(x)$
e	1	–0.5321 – 4.597i
1	0.5671	–1.534 – 4.375i
–1/e	–1	–1
–1/4	–0.3574	–2.153

3.6 GALILEAN TRANSFORMATION

In order to make use of Lambert *W*-function for our specific purpose, let's look for the general solution of an implicit equation given by

$$p^{ax+b} = cx + d \tag{16}$$

If equation (16) can be expressed in the form $tp^t = R$, then solution involving *W*-function is obtained as follows:

$$tp^t = R \tag{17a}$$

$$\Rightarrow te^{t\ln p} \ln p = R\ln p$$

$$\Rightarrow t = \frac{1}{\ln p} Lambert\ \ W(R\ln p) \tag{17b}$$

Without the loss of generality, to express equation (16) to the form of equation (17a), the following Galilean transformation is made.

$$t = mx + n \tag{18a}$$

Thus, equation (16) becomes:

$$\Rightarrow \frac{m}{c} p^{b-\frac{an}{m}} = tp^{-\frac{at}{m}} + \left(d - \frac{nc}{m}\right) p^{-\frac{at}{m}} \frac{m}{c} \tag{18b}$$

Equation (18b) will reduce to equation (17a) if

$$m = -a \text{ and } d - \frac{nc}{m} = 0; \ \Rightarrow n = \frac{md}{c}$$

Thus, the required **Galilean transformation** is

$$t = -ax - \frac{ad}{c} \tag{18c}$$

Hence, equation (18b) is reduced to the following form

$$tp^t = R \text{ where } R = -\frac{a}{c} p^{b-\frac{ad}{c}}$$

The solution of this equation is given by equation (17b) i.e.

$$\Rightarrow -ax - \frac{ad}{c} = \frac{1}{\ln p} Lambert\ \ W\left[-\frac{a}{c} p^{b-\frac{ad}{c}} \ln p\right]$$

$$x = -\frac{1}{a\ln p} Lambert\ \ W\left[-\frac{a\ln p}{c} p^{b-\frac{ad}{c}}\right] - \frac{d}{c} \tag{18d}$$

Equation (18d) is the exact and explicit solution of the implicit equation (16). Thus, the exact and explicit form of time of flight of dust particles as given by equation (14) in terms of Lambert *W*-function can be written, using equation (18d), as follows:

$$t_2 = \frac{m}{b}\,Lambert\;\; W\left[-\left(1+\frac{b}{mg}u_y\right)e^{-\left(1+\frac{b}{mg}u_y\right)}\right]+\frac{1}{g}\left(u_y+\frac{mg}{b}\right) \tag{19}$$

$$t_2 = \frac{m}{b}\,Lambert\;\; W(x)+\frac{1}{g}\left(u_y+\frac{mg}{b}\right)$$

Where $$x = -\left(1+\frac{b}{mg}u_y\right)\exp\left\{-\left(1+\frac{b}{mg}u_y\right)\right\}$$

3.7 HORIZONTAL RANGE

Horizontal range R can be evaluated using either of the following definitions:

$R = x(t_2)$; or range R is the value of x for which $y(x) = 0$.

It is convenient to determine range R using the second definition i.e. using the trajectory equation (11b). Therefore, equation (11b) reduces to

$$\frac{1}{u_x}\left(u_y+\frac{mg}{b}\right)x+\frac{m^2g}{b^2}\ln\left(1-\frac{bx}{mu_x}\right)=0$$

$$\Rightarrow Exp\left\{-\frac{b^2}{m^2g}\frac{1}{u_x}\left(u_y+\frac{mg}{b}\right)R\right\}=-\frac{b}{mu_x}R+1 \tag{20}$$

Equation (20) is again a transcendental equation whose explicit solution can be written in terms of Lambert *W*-function only. Thus, the horizontal range R is obtained as follows using equation (18d)

$$R = \frac{m^2gu_x}{b^2\left(u_y+\frac{mg}{b}\right)}\,Lambert\;\; W\left[-\left(1+\frac{bu_y}{mg}\right)e^{-\left(1+\frac{bu_y}{mg}\right)}\right]+\frac{mu_x}{b}$$

$$R = \frac{m^2gu_x}{b^2\left(u_y+\frac{mg}{b}\right)}\,Lambert\;\; W(x)+\frac{mu_x}{b} \tag{21}$$

Where $$x = -\left(1+\frac{b}{mg}u_y\right)\exp\left\{-\left(1+\frac{b}{mg}u_y\right)\right\}$$

4. CONTEXT OF ROVER OPERATIONS

Having arrived at the general formalism of dust dynamics, now let's apply it to study the dynamics of dust specific to the following modes of rover operations as discussed in preceding sections.

4.1 NORMAL ROVER OPERATION

As derived in equation (3), in this mode of rover operations, we have the following component velocities:

$$u_x = V_{CM}(1-\cos\theta) \text{ and } u_y = V_{CM}\sin\theta \tag{22a}$$

Thus, substituting equation (22a) in equations (12), (13b), (19), and (21), we obtain the following set of equations which govern the dynamics of dust particles kicked off the rover wheel:

$$t_1 = \frac{m}{b}\ln\left(1+\frac{b}{mg}V_{CM}\sin\theta\right) \tag{22b}$$

$$H = -g\left(\frac{m}{b}\right)^2\ln\left(1+\frac{b}{mg}V_{CM}\sin\theta\right)$$
$$+\frac{m}{b}\left(V_{CM}\sin\theta+\frac{mg}{b}\right)\left[1-\exp\left\{-\ln\left(1+\frac{b}{mg}V_{CM}\sin\theta\right)\right\}\right] \tag{22c}$$

$$t_2 = \frac{m}{b}\,Lambert\;\; W\left[\begin{array}{l}-\left(1+\frac{bV_{CM}\sin\theta}{mg}\right)\\ *\exp\left\{-\left(1+\frac{bV_{CM}\sin\theta}{mg}\right)\right\}\end{array}\right]$$
$$+\frac{1}{g}\left(V_{CM}\sin\theta+\frac{mg}{b}\right) \tag{22d}$$

$$R = \frac{m^2gV_{CM}(1-\cos\theta)}{b^2\left(V_{CM}\sin\theta+\frac{mg}{b}\right)}\,Lambert\;\; W\left[\begin{array}{l}-\left(1+\frac{bV_{CM}\sin\theta}{mg}\right)\\ *\exp\left\{-\left(1+\frac{bV_{CM}\sin\theta}{mg}\right)\right\}\end{array}\right]$$
$$+\frac{m}{b}V_{CM}(1-\cos\theta) \tag{22e}$$

4.2 CONTINGENCY ROVER OPERATION

In this mode of rover operation, as derived in equation (5), we have the following component velocities:

$$u_x = \omega R\cos\theta \text{ and } u_y = \omega R\sin\theta \tag{23a}$$

Thus, substituting equation (23a) in equations (12), (13b), (19), and (21), we obtain the following set of equations which govern the dynamics of dust particles kicked off the rover wheel:

$$t_1 = \frac{m}{b}\ln\left(1+\frac{b}{mg}\omega R\sin\theta\right) \tag{23b}$$

$$H = -g\left(\frac{m}{b}\right)^2 \ln\left(1+\frac{b}{mg}\omega R\sin\theta\right)$$

$$+\frac{m}{b}\left(\omega R\sin\theta+\frac{mg}{b}\right)\left[1-\exp\left\{-\ln\left(1+\frac{b}{mg}\omega R\sin\theta\right)\right\}\right] \quad (23c)$$

$$t_2 = \frac{m}{b}LambertW\left[-\left(1+\frac{b\omega R\sin\theta}{mg}\right)\exp\left\{-\left(1+\frac{b\omega R\sin\theta}{mg}\right)\right\}\right]$$

$$+\frac{1}{g}\left(\omega R\sin\theta+\frac{mg}{b}\right) \quad (23d)$$

$$R = \frac{m^2 g\omega R\cos\theta}{b^2\left(\omega R\sin\theta+\frac{mg}{b}\right)}Lambert\ W\left[\begin{array}{l}-\left(1+\frac{b\omega R\sin\theta}{mg}\right)\\ *\exp\left\{-\left(1+\frac{b\omega R\sin\theta}{mg}\right)\right\}\end{array}\right]$$

$$+\frac{m}{b}\omega R\cos\theta \quad (23e)$$

5. EFFECTS OF LUNAR TRACE-GASES ATMOSPHERE

As discussed above that the non-zero gravity of the Moon is manifested to a non-vacuum atmosphere consisting of trace gases. In order to see the effects of these local trace gases near the lunar surface, we have to see the Taylor series form of equations (12), (13b), (19), and (21). We have the time of ascend t_1 as given by equation (12):

$$t_1 = \frac{m}{b}\ln\left(1+\frac{b}{mg}u_y\right) = \frac{m}{b}\ln(1+\varepsilon) \quad \text{where } \varepsilon = \frac{b}{mg}u_y \text{ and}$$

$b \le 10^{-3}$ *gm / sec* for a lunar particle of diameter ≤ 1 *mm*.

Using Taylor series expansion, we get

$$t_1 = \frac{m}{b}\left[\varepsilon-\frac{\varepsilon^2}{2}+\frac{\varepsilon^3}{3}-\frac{\varepsilon^4}{4}+.....\right]$$

$$\Rightarrow t_1 = \frac{m}{b}\varepsilon\left[1-\frac{\varepsilon}{2}+\frac{\varepsilon^2}{3}-\frac{\varepsilon^3}{4}+.....\right]$$

$$\Rightarrow t_1 = \frac{u_y}{g}\left[1-\frac{1}{2}\left(\frac{b}{m}\right)\left(\frac{u_y}{g}\right)+O\left(\varepsilon^2\right)\right] \quad (24a)$$

Since $t_{1vac} = \frac{u_y}{g}$ is the time of ascend of dust particles in vacuum, equation (24a) reduces to

$$\Rightarrow t_1 = t_{1vac}\left[1-\frac{1}{2}\left(\frac{b}{m}\right)t_{1vac}+O\left(\varepsilon^2\right)\right] \quad (24b)$$

Thus, we see from equation (24b) that trace gases in lunar atmosphere decrease the time of ascend of dust particles as expected.

Substituting $\varepsilon = \frac{b}{mg}u_y$ in the equation (13b) for maximum height H and expanding in Taylor series form, we get

$$H = -\frac{m^2 g}{b^2}\ln(1+\varepsilon)+\frac{m}{b}\left(u_y+\frac{mg}{b}\right)$$

$$*\left[1-\left\{1-\ln(1+\varepsilon)+\frac{1}{2}\ln(1+\varepsilon)\ln(1+\varepsilon)-.....\right\}\right]$$

$$H = \ln(1+\varepsilon)\left[-\frac{m^2 g}{b^2}+\frac{m}{b}\left(u_y+\frac{mg}{b}\right)\left\{1-\frac{1}{2}\left(\varepsilon-\frac{\varepsilon^2}{2}+..\right)\right\}\right]$$

$$H = \left[\varepsilon-\frac{\varepsilon^2}{2}+.....\right]\left[\begin{array}{l}\frac{mu_y}{b}-\frac{1}{2}\frac{b}{mg}u_y\frac{m}{b}u_y-\frac{1}{2}\frac{bu_y}{mg}\frac{m}{b}\frac{mg}{b}\\ +\frac{1}{4}\varepsilon^2\frac{m}{b}\left(u_y+\frac{mg}{b}\right)+....\end{array}\right]$$

$$H = \varepsilon\left[\frac{1}{2}\frac{mu_y}{b}-\frac{1}{2}\frac{u_y^2}{g}+\frac{1}{4}\varepsilon^2\frac{m}{b}\left(u_y+\frac{mg}{b}\right)+....\right]$$

$$-\frac{1}{2}\varepsilon^2\left[\frac{1}{2}\frac{mu_y}{b}-\frac{1}{2}\frac{u_y^2}{g}+\frac{1}{4}\varepsilon^2\frac{m}{b}\left(u_y+\frac{mg}{b}\right)+....\right]$$

$$H = \frac{u_y^2}{2g}-\frac{b}{m}\frac{u_y}{g}\frac{u_y^2}{2g}+\frac{1}{4}\frac{b^2}{m^2}\frac{u_y^4}{g^3}+\frac{1}{4}\frac{b}{m}\frac{u_y^3}{g^2}-\frac{1}{4}\frac{b}{m}\frac{u_y^3}{g^2}$$

$$+\frac{1}{4}\frac{b^2}{m^2}\frac{u_y^4}{g^3}+O\left(\varepsilon^4\right)$$

$$H = \frac{u_y^2}{2g}\left[1-\frac{b}{m}\frac{u_y}{g}+\frac{b^2}{m^2}\left(\frac{u_y}{g}\right)^2+O\left(\varepsilon^4\right)\right]$$

$$H = H_{vac}\left[1-\frac{b}{m}t_{1vac}+\frac{b^2}{m^2}t_{1vac}^2+O\left(\varepsilon^4\right)\right] \quad (25a)$$

$$H = H_{vac}\left[1-\frac{b}{m}t_{1vac}\right] \quad (25b)$$

Thus, it is seen from equation (25) that presence of trace gases in lunar atmosphere decreases the maximum height attained by dust particles.

Total time of flight t_2 is obtained from equation (19). One can expand the *W*-function in series using the series expansion of *W*-function. However, just to see

the first order effect, the Taylor series form of time of flight as given by equation (14) may be used.

$$\Rightarrow 1-\frac{b}{m}t_2+\frac{1}{2}\frac{b^2}{m^2}t_2^2+O\left(t_2^3\right)=-\frac{g}{\left(u_y+\frac{mg}{b}\right)}t_2+1$$

$$\Rightarrow t_2\left[\frac{1}{2}\frac{b^2}{m^2}t_2+\frac{g}{\left(u_y+\frac{mg}{b}\right)}-\frac{b}{m}\right]=0 \qquad (26a)$$

The non-trivial solution of equation (26a) is given by

$$\Rightarrow t_2=\frac{2m^2}{b^2}\left[\frac{b}{m}-\frac{g}{\left(u_y+\frac{mg}{b}\right)}\right]=\frac{2u_y}{g}\frac{1}{\left[1+\frac{b}{m}\frac{u_y}{g}\right]}$$

$$\Rightarrow t_2=\frac{t_{2vac}}{1+\frac{b}{m}t_{1vac}} \qquad (26b)$$

Thus, we see from equation (26b) that the total time of flight of dust particles also decreases because of the presence of drag force due to lunar trace gases.

Again, using the same analogy as in the time of flight, we take Taylor series expansion of the equation (20).

$$\Rightarrow 1-\frac{b^2}{m^2g}\frac{1}{u_x}\left(u_y+\frac{mg}{b}\right)R+\frac{1}{2}\frac{b^4}{m^4g^2u_x^2}\left(u_y+\frac{mg}{b}\right)^2R^2+.....$$

$$=-\frac{b}{mu_x}R+1$$

$$\Rightarrow -\frac{b^2}{m^2g}\frac{R}{u_x}\left(u_y+\frac{mg}{b}\right)+\frac{bR}{mu_x}+\frac{1}{2}\frac{b^4}{m^4g^2u_x^2}\left(u_y+\frac{mg}{b}\right)^2R^2=0$$

$$\Rightarrow \frac{b^2}{m^2gu_x}R\left[-u_y+\frac{1}{2m^2gu_x}\left(bu_y+mg\right)^2R\right]=0 \qquad (27a)$$

The non-trivial solution of equation (27a) is given by

$$\Rightarrow -u_y+\frac{1}{2m^2gu_x}\left(bu_y+mg\right)^2R=0$$

$$R=\frac{2m^2gu_xu_y}{\left[bu_y+mg\right]^2}=\frac{2m^2gu_xu_y}{m^2g^2\left[1+\frac{bu_y}{mg}\right]^2}=\frac{2u_xu_y}{g}\frac{1}{\left[1+\frac{b}{m}\frac{u_y}{g}\right]^2}$$

$$\Rightarrow R=\frac{R_{vac}}{\left[1+\frac{b}{m}t_{1vac}\right]^2} \qquad (27b)$$

Hence, we see from equation (27b) that horizontal range of the dust particles decreases as expected.

Thus, presence of trace gases in the near-surface lunar atmosphere due to non-zero gravity of the Moon modifies the dynamics of dust during rover operations and the worst case of the dynamics is given by the ideal vacuum conditions of Moon i.e. considering as if Moon does not have an atmosphere at all. Thus, following results are applicable.

The maximum height attained by dust particles during rover operations: $H_{\max}=\frac{u_y^2}{2g}$ (28a)

The maximum range of dust particles:

$$R_{\max}=\frac{2u_xu_y}{g} \qquad (28b)$$

The total time of flight of dust particles:

$$T_{\max}=\frac{2u_y}{g} \qquad (28c)$$

6. MINIMUM VELOCITY OF WHEEL TO BREAK A DUST PARTICLE OFF THE WHEEL

Assuming that a dust particle of mass m and radius r sticks to the rover wheel of mass M and radius R at the point of contact A and it is transported at the point P traversing a distance arc AP during time t. Also, let's assume that dust particles adhere to the surface of wheel with a force 'F' and the co-efficient of friction between the wheel and dust particles is μ. Let I be moment of inertia of wheel, F_r be the radial force and N be the normal reaction.

If V and ω be the linear velocity and angular velocity of centre of mass (CM) of the wheel before picking up a dust particle. After a dust particle gets stuck to the surface of wheel, angular velocity of the centre of mass will be changed to ω_1 and V becomes V_1. In ideal situations, rolling motion of wheel on undulating dusty surface will cause continuous variation of CM in vertical direction. However, for simplicity in the calculation, this is neglected and the position of CM of wheel is assumed to be at a constant height from the ground. Frictional force of the ground surface will try to cease the rolling motion of the wheel after some time. However, the force of traction compensates this frictional force and maintains uniform angular velocity

of the CM. Moreover, since we are interested in the dynamics of dust, we consider only the forces acting on the dust at the instantaneous position P, and we neglect such kind of forces.

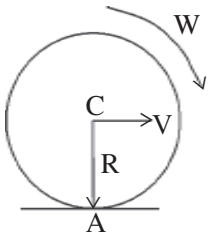

Fig.: 4a

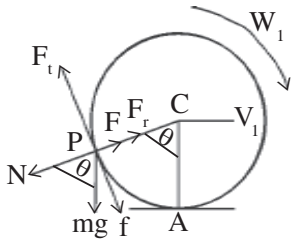

Fig.: 4b

Defining the axes as shown in figure 4b, we can write down the equations of motion of dust particle at P as follows:

$$\frac{mV_1^2}{R+r} = F - N - mg\cos\theta \tag{29a}$$

$$ma_t = mg\sin\theta + f \tag{29b}$$

where a_t is the tangential acceleration of dust particle and f is the frictional force between wheel and dust particle.

Applying work-energy theorem, we get

Work done by all forces = Change in kinetic energy

$$W_F + W_N + W_{mg} + W_{Mg} + W_f = K_f - K_i \tag{30}$$

The non-zero work done is obtained as follows:

Work done by gravity acting on mass m

$$W_{mg} = \int_0^\theta m\vec{g}.d\vec{s} = \int_0^\theta mg\sin\theta d(-R\theta) = -mgR(1-\cos\theta) \tag{31a}$$

Work done by friction f

$$W_f = \int_0^\theta \mu\vec{N}.d\vec{s} = \mu NR\cos\pi\int_0^\theta d\theta = -\mu NR\theta \tag{31b}$$

Initial Kinetic energy K_i

$$K_i = \frac{1}{2}m\left|\vec{V}+\vec{\omega}\times\vec{R}\right|^2 + \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 \tag{32a}$$

Final Kinetic energy K_f

$$K_f = \frac{1}{2}m\left|\vec{V_t}+\vec{\omega_1}\times\vec{R}\right|^2 + \frac{1}{2}MV_1^2 + \frac{1}{2}I\omega_1^2 \tag{32b}$$

For pure rolling, $V = \omega R$ & $V_t = \omega_1 R = V_1$ thus equation (32) becomes

$$K_i = \frac{1}{2}m(V-\omega R)^2 + \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MV^2 + \frac{1}{2}I\frac{V^2}{R^2} \tag{33a}$$

$$K_f = \frac{1}{2}m*2V_1^2(1-\cos\theta) + \frac{1}{2}MV_1^2 + \frac{1}{2}I\frac{V_1^2}{R^2} \tag{33b}$$

Using equation (31) and (33), equation (30) becomes,

$$\Rightarrow V_1^2\left[2m(1-\cos\theta) + M + \frac{I}{R^2}\right]$$

$$= V^2\left[M + \frac{I}{R^2}\right] - 2mgR(1-\cos\theta) - 2\mu NR\theta \tag{34a}$$

Substituting the value of V_1 from equation (29a), equation (34a) becomes

$$V^2 = \left\{\begin{aligned}&\left[\frac{(R+r)F}{m} - \frac{N(R+r)}{m} - g(R+r)\cos\theta\right]\\ &*\left[2m(1-\cos\theta) + M + \frac{I}{R^2}\right] + 2mgR(1-\cos\theta) + 2\mu NR\theta\end{aligned}\right\}$$

$$\div\left[M + \frac{I}{R^2}\right] \tag{35a}$$

At the point of BREAK-OFF, N = 0; thus equation (35a) reduces to

$$V = \sqrt{\left\{\begin{aligned}&\left[\frac{(R+r)F}{m} - g(R+r)\cos\theta\right]\\ &*\left[2m(1-\cos\theta) + M + \frac{I}{R^2}\right] + 2mgR(1-\cos\theta)\end{aligned}\right\}\div\left[M + \frac{I}{R^2}\right]} \tag{35b}$$

Equation (35b) defines the required minimum velocity "V" of rover wheel, to break a dust particle off the wheel at an angle "θ" during normal rover operations.

7. RESULTS AND DISCUSSIONS

Having derived the complete set of equations from the proposition which governs the dynamics of dust during rover operations, it can be now applied to four-wheeled Chandrayaan-2 rover of 18 kg mass (on the Earth).

The rover is planned to land on a vast plain area near lunar South pole at 88°S latitude. The lunar surface temperature in and around the landing site latitude varies between –120°C to –160°C [13-17]. The rover has a solar panel at a height of minimum of 75 cm from ground and rover is configured such that $V_{CM} = \omega R$ for all modes of operation. Considering diameter of a dust particle as 50 μm and corresponding mass of a dust particle as 1 μg [3], the calculations are carried out. Since the properties of lunar near-surface atmosphere are not known, calculation is carried out for each gas component of atmosphere

whose property is nearly found. Though the dynamic viscosity of gases η_d varies with temperature and pressure, its variation with pressure is very weak and hence is not considered. However, the variation of with temperature is calculated using Sutherland's model by which varies with absolute temperature T as follows:

$$\eta_d = \eta_o \frac{T_o + S}{T + S}\left(\frac{T}{T_o}\right)^{\frac{3}{2}} \tag{36}$$

in which η_0 is the viscosity at the reference temperature T_0 and S is the Sutherland constant. Thus, for helium and hydrogen gases of known Sutherland constants, values are calculated using equation (36) at the landing site temperature of –120°C. However, argon gas of unknown Sutherland constant, available value at –100°C is being used [18]. Though Chandrayaan-2 rover is designed for a nominal speed of 1 cm/sec, analysis is carried out for three different speeds and results are tabulated in the following tables (Table 2, 3, and 4).

Also, it can be calculated that speed of rover required to throw dust particles up the solar panel height > 155 cm/s. Again, considering a dust particle of diameter 1mm, mass 1mg, and rover of each wheel mass 500 gm, diameter 180 mm, it can be shown that minimum speed required to break a dust particle off the rover wheel > 5 m/s.

Thus, a rover moving with a speed of 1 cm/s can not break dust off the wheel during its operations. On the other hand too; supposing that dust breaks off the wheel, the dust can not clear the wheel diameter because maximum height attained by dust particles (from the instantaneous point of break-off) << 1 cm.

Table 2a: Normal operations at V_{CM} = 1 cm/sec

Trace Gas	b (10^{-3} gm/sec)	x	$W_0(x)$	$W_{-1}(x)$	t_2 (drag) (sec)	t_2 (vacuum) (sec)	R (drag) (cm)	R (vacuum) (cm)
He	0.011402	-0.3262736800	-0.586591	-1.573009	0.008651	0.010051	0.003154	0.005764
Ar	0.013177	-0.3153490738	-0.542491	-1.662243	0.008497	0.010051	0.002932	0.005764
H_2	0.005048	-0.3578656543	-0.783090	-1.253684	0.009322	0.010051	0.004265	0.005764

Table 2b: Normal operations at V_{CM} = 1 cm/sec

Trace Gas	b (10^{-3} gm/sec)	t_1 (drag) (sec)	t_1 (vacuum) (sec)	H (drag) (cm)	H (vacuum) (cm)
He	0.011402	0.003973	0.005025473	0.001504760	0.002058313
Ar	0.013177	0.003856	0.005025473	0.001446236	0.002058313
H_2	0.005048	0.004479	0.005025473	0.001765332	0.002058313

Table 3a: Normal operations at 10 cm/sec.

Trace Gas	b (10^{-3} gm/sec)	x	$W_0(x)$	$W_{-1}(x)$	t_2 (drag) (sec)	t_2 (vacuum) (sec)	R (drag) (cm)	R (vacuum) (cm)
He	0.011402	-0.0080385746	-0.008104	-6.730092	0.058954	0.100509	0.050243	0.576498
Ar	0.013177	-0.0037300475	-0.003744	-7.622429	0.057815	0.100509	0.043504	0.576498
H_2	0.005048	-0.1029401015	-0.115549	-3.536842	0.067776	0.100509	0.109913	0.576498

Table 3b: Normal operations at 10 cm/sec

Trace Gas	b (10^{-3} gm/sec)	t_1 (drag) (sec)	t_1 (vacuum) (sec)	H (drag) (cm)	H (vacuum) (cm)
He	0.011402	0.016721	0.050254727	0.047937975	0.20583131
Ar	0.013177	0.015413	0.050254727	0.043096864	0.20583131
H_2	0.005048	0.025025	0.050254727	0.081468601	0.20583131

Table 4a: Normal operations at 20 cm/sec

Trace Gas	b (10^{-3} gm/sec)	x	$W_0(x)$	$W_{-1}(x)$	t_2 (drag) (sec)	t_2 (vacuum) (sec)	R (drag) (cm)	R (vacuum) (cm)
He	0.011402	-0.0000483209	-0.000048	-12.46018	0.109279	0.201018	0.100608	2.305994
Ar	0.013177	-0.0000092725	-0.000009	-14.24485	0.108098	0.201018	0.087052	2.305994
H_2	0.005048	-0.0139857096	-0.014186	-6.073685	0.120318	0.201018	0.227248	2.305994

Table 4b: Normal operations at 20 cm/sec

Trace Gas	b (10^{-3} gm/sec)	t_1 (drag) (sec)	t_1 (vacuum) (sec)	H (drag) (cm)	H (vacuum) (cm)
He	0.011402	0.035736	0.100509453	0.112057611	0.82332524
Ar	0.013177	0.020158	0.100509453	0.099389309	0.82332524
H_2	0.005048	0.035736	0.100509453	0.209153406	0.82332524

8. CONCLUSIONS

Extremely fine lunar dust clings tenaciously to surfaces making the removal efforts quite difficult. Deposition of such dust on sensitive surfaces like solar panel, optical lenses, etc. causes serious problems. Thus, complete understanding of the dynamics of dust during rover operations is necessary. However, the non-zero gravity of the Moon is manifested as a non-vacuum atmosphere near the lunar surface consisting of trace gases making determination of trajectory of dust difficult. A proposition is arrived to define the equations which best describe these dynamics by calling for a linear drag force in the equations of motion which subsequently leads to transcendental equations for which exact closed-form solutions can be provided with Lambert *W*-function only. However, worst case conditions are met considering as if Moon does have an ideal vacuum atmosphere. An analysis taking Chandrayaan-2 rover is presented and it shows that the effect of probable resistance offered by near-surface lunar atmosphere consisting of an equilibrium trace gases is unnoticeable for rovers operating at low speed and the effect appears as the rover speed increases as expected. It is envisaged that the proposed model may also be used for Mars rovers as first order approximation.

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